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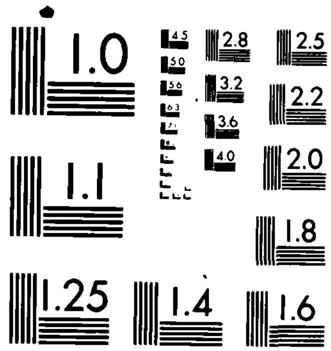
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Improvement Curves, Production Rate,
And Optimal Contractor Behavior

by

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ABSTRACT

There has been much interest in recent years in the relationships among learning, production rate, and program costs. These relationships are of particular interest in military acquisitions research where, because of the nature of the funding process, the government must assess the cost impact of numerous production rate changes. One approach that is often used to analyze the problem is an empirical application of the Alchian cost function. This ~~research~~ provides constructive criticism of the economic analyses that are often applied to this very difficult managerial problem.

Keywords: Cost analyses, weapon systems, cost estimates,

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INTRODUCTION

There has been much interest in recent years in the relationships among learning, production rate, and program costs. These relationships are of particular interest in the military acquisition of made-to-order equipment. At the outset of a weapon system program, a tentative monthly production schedule for the life of the program is negotiated between the contracting parties. This planning schedule covers the life of the program, but formal contractual agreements between the Department of Defense and manufacturers usually cover only annual delivery requirements. Since annual funding allocations are characterized by political uncertainties, there is often a need to deviate from the planned production rate during the production phase of the program. Coincident with these rate changes, new cost estimates are required to support contract negotiations and additional funding requests.

There are many proposed methodologies for assessing the cost impact of a production rate change. One of the more popular approaches is an application of the Alchian [1, 2] cost function. Even within this framework there is very little agreement about the relationships among learning, production rate, and program cost. While some studies, for example, Gulledge, Womer, Tarimcilar [11] and Womer and Gulledge [15] make assumptions concerning the cost impact of the above factors in developing models of the optimal contractor behavior, others (e.g., Smith [14], Large, et. al [12], Bemis [3, 4], Cox and Gansler [8], Crouch [9], Cox, et. al. [7], Bohn and Kratz [6]) address the problem directly by attempting to statistically estimate the above influences. In these latter studies no attempt is made to model contractor behavior in the analytical relations. The purpose of this paper is to illustrate the problems that follow from ignoring contractor behavior when developing models of production programs.

ESTIMATING COST IMPACTS

The learning curve, first formulated by Wright [17], is an empirically specified relationship that yields declining unit costs with increases in cumulative output. In recent years the more commonly used terminology has been "improvement curve." The improvement curve allows for reductions in cost that are due to other factors in addition to repetition (learning). Gold [10] includes changes in product design, product mix, technology, facilities, etc. in this listing of other factors. Learning and improvement curves are described mathematically as

$$Z = \beta_0 X_{1t}^{\beta_1} \tag{1}$$

where

Z = the average cost of the units produced during some period of time t,

β_0 = a constant, commonly called the first unit cost,

β_1 = a parameter describing the slope of the quantity/cost curve,

X_{1t} = cumulative quantity produced through time t.

Studies attempting to ascertain the relationships among production rate, learning, and program costs often use the following augmented model [2, 3, 4, 7, 8, 9, 12, 14]:

$$Z_t = \beta_0 X_{1t}^{\beta 1} X_{2t}^{\beta 2}$$

where

X_{2t} = some measure (usually a proxy) for production rate during time period t,

β_2 = a parameter describing the slope of the rate/cost curve.

Some researchers (e.g., Bohn and Kratz [6]) call equation (2) the "rate analysis curve model."

The parameters in equation (2) are estimated from the log-linear form of the relationship using linear regression or directly from (2) using nonlinear regression. Unfortunately both of these techniques often are plagued with statistical problems due to the collinearity between the independent variables, X_1 and X_2. The source of this collinearity may be reasoned as follows. Often made-to-order production programs are characterized by initial production at a low rate with a gradual buildup in production rate throughout the program. In fact, given a learning curve, if the resource use rate does not decline, production rate must increase during the program. As a result sequential observations on cumulative quantity can be highly correlated with production rate. This probably explains the conclusion by Large, et. al. [12], that the influence of production rate could not be estimated with confidence. More recent studies have also been unable to significantly measure the influence of production rate. Both positive and negative estimates of β_2, the slope of the rate/cost curve, have been obtained. Assuming that an increase in rate requires an increase in resources, a positive slope for the rate/cost curve implies decreasing returns to the variable resources. That is, an increase in production rate causes an increase in required resources (and hence unit cost). A negative slope implies increasing returns since an increase in rate requiring an increase in resources decreases unit cost.

There are many applications where estimates of the parameters in equation (2) are provided. For example, Bemis [3] provides a table of estimates for many defense items. The estimated values for the quantity slope (β_1) and the rate slope (β_2) are presented in Table 1. Notice that all of the estimated values for β_2 are less than zero. The estimates for tactical missile programs presented by Cox and Gansler [8] are presented in Table 2. While the Bullpup and Tow estimates seem reasonable, the Sparrow and Sidewinder estimates have $\beta_2 < 0$.

Table 1. Values of β_1 and β_2 estimated by Bemis [3].

System	β_1	β_2
Aircraft A	-.4521	-.0365
Aircraft C	-.1959	-.3310
Aircraft D	-.1811	-.5564
Aircraft E	-.1440	-.5735
Aircraft F	-.2076	-.8034
Aircraft G	-.2515	-.2969
Aircraft H	-.4266	-.1297
Helicopter	-.2550	-.1633
Jet Engine A	-.4150	-.1203

Table 1 (continued)

System	β_1	β_2
Jet Engine B	-.4860	-.1600
Missile G&C	-.1219	-.7515
Ordnance Item A	-.1828	-.0439
Radar Set A	-.1031	-.1714
Radar Set B	-.0160	-.1408

Note: Six additional items were included in Bemis' data summary. These were not included because in four cases the rate slope was not provided, and in two cases the quantity slope was not provided.

Table 2. Estimates of β_1 and β_2 for missile programs as presented by Cox and Gansler

System	β_2	β_1
Sparrow (1st source)	-.0218	-.2413
Sparrow (2nd source)	-.1156	-.1943
Bullpup	.0058	-.2810
Tow	.0101	-.0130
Sidewinder	-.2881	-.0663

As discussed by Cox and Gansler [8], different signs for β_2, even if statistically significant, are not necessarily contradicting. In the short-run, both increasing and decreasing returns can exist. Furthermore, even if the data indicate falling unit cost as rate increases, this does not necessarily imply increasing returns to the variable factors. The firm could be producing in the region of diminishing returns on the short-run cost surface, but the dominating learning (cumulative quantity) effect could be causing unit costs to decline.

There are also additional problems with the formulation described by equation (2). Cox and Gansler [8] and Bohn and Kratz [6] use lot size as a proxy for production rate. However, the time required to produce a lot often changes over the program's life. This is true in much of the data that these authors have analyzed, namely the C141 airframe program, the F102 airframe program, the Black Hawk helicopter program, and the F4 airframe program. For example, lot sizes of 15 and 20 are not good proxies for production rate if the time horizons for the two lots are 12 and 16 months respectively.

It is now possible to precisely state the purpose of this paper. As noted, many researchers [2, 3, 4, 6, 7, 8, 9, 12, 14] have examined the model presented in equation (2). These models are currently being used by the military; e.g., the model presented in [6] is currently being used by the U.S. Air Force Systems Command for planning purposes. We will use this same model, combined with an assumption about contractor behavior, to show that some of the parameter estimates that have been obtained for equation (2) imply that the contractor should produce all units at the very end of the production program. We also note that if the parameters of equation (2) are estimated from models which include optimal contractor behavior (as in [15]), the resulting estimates are consistent with actual observed behavior.

CONTRACTOR BEHAVIOR

Elsewhere [11] we have analyzed a model of the same form as equation (2) with the assumption of decreasing returns to the variable resources; i.e., $\beta_2 > 0$. Here we show that if $\beta_2 \leq 0$, then in the presence of learning, $0<\beta_1<1$, optimal contractor behavior results in producing the output for the entire program either at the beginning or at the end of the production time period. Since this is inconsistent with observed and logical behavior, we conclude that β_2 must be positive if the model at equation (2) is to be relevant.

This proof does not imply that our model is right and the practitioners are wrong. In fact, we are using the practitioner's model. Our point is the following: When using models like equation (2), there are appropriate modeling procedures that will yield parameter estimates that fall within the admissible ranges (see reference [15]).

Since Z_t is the average unit cost for period t, discounted program cost may be expressed as

$$C = \sum_{t=1}^{T} Z_t X_{2t}/(1+r)^t,$$

$$= \sum_{t=1}^{T} \beta_0 X_{1t}^{\beta_1} X_{2t}^{\beta_2+1}/(1+r)^t \tag{3}$$

where r is a discount rate. The variables of equation (3) are connected by the stage transformation functions

$$X_{1t-1} = X_{1t} - X_{2t} \qquad t = 1, \ldots, T \tag{4}$$

and the boundary conditions

$$X_{10} = 0, \tag{5}$$

and

$$X_{1T} = V. \tag{6}$$

For major weapon systems programs, the wording of the contract is an important determinant of firm behavior. The assumption is that the contract is structured so as to induce cost minimizing behavior by the contractor. This might be in the form of either a firm fixed price contract, cost-plus incentive fee, or award fee contract. Cost minimization is not automatic with these contracts; see Boger, Jones and Sontheimer [5] for a discussion of the problems of constructing contracts with incentives for cost minimization. In particular we assume that the contractor is motivated to minimize discounted contract cost by choosing production rate in each of the time periods. The contractor's problem is

$$\text{Minimize} \quad C = \sum_{t=1}^{T} \beta_0 X_{1t}^{\beta_1} X_{2t}^{\beta_2+1}/(1+r)^t$$

subject to:

$$X_{10} = 0, \tag{7}$$

$$X_{1T} = V,$$

a problem in dynamic programming. This problem is of the same form as that analyzed in reference [10].

There we show how to solve the problem if β_2 is greater than zero. If $-1<\beta_2<0$ as estimated for almost all the systems in Tables 1 and 2, then the solution to (7) takes a particularly simple but unrealistic form. The dynamic programming problem may be stated as a sequence of static optimization problems using the recursion equations of dynamic programming as presented by Nemhauser [13]. In this case they are

$$f_1\ (X_{11}) = \min_{X_{21}=X_{11}} \beta_0 X_{11}^{\beta_1} X_{21}^{\beta_2+1}/(1+r), \tag{8}$$

$$f_t\ (X_{1t}) = \min_{0\le X_{2t}\le X_{1t}} \beta_0 X_{1t}^{\beta_1} X_{2t}^{\beta_2+1}/(1+r)^t, \tag{9}$$

for $t = 2, 3, \ldots, T$ with $X_{1T} = V$. The problem at equation (8) is solved as

$$f_1^*\ (X_{11}) = \beta_0 X_{11}^{\beta_1+\beta_2+1}/(1+r). \tag{10}$$

Therefore the problem at stage 2 is

$$f_2\ (X_{12}) = \min_{0\le X_{22}\le X_{12}} \beta_0 X_{12}^{\beta_1} X_{22}^{\beta_2+1}/(1+r)^2 \tag{11}$$

$$+ \beta_0 (X_{12}-X_{22})^{\beta_1+\beta_2+1}/(1+r).$$

Differentiating with respect to X_{22} yields

$$\frac{df_2}{dX_{22}} = \beta_0(\beta_2+1) X_{12}^{\beta_1} X_{22}^{\beta_2}/(1+r)^2 \tag{12}$$

$$- \beta_0(\beta_1+\beta_2+1)(X_{12}-X_{22})^{\beta_1+\beta_2}/(1+r).$$

If $\beta_1+\beta_2+1 \le 0$, as is the case for Aircraft F in Table 1, then the derivative is positive over the entire range $0\le X_{22}\le X_{12}$. In this case the solution to equation (11) is

$X_{22}=0$; that is, no production in period 2. If $\beta_1+\beta_2+1>0$ then equation (11) equals zero in the interval and the second-order conditions must be examined.

The second derivative is

$$\frac{d^2f_2}{dX_{22}^2} = \beta_0(\beta_2+1)\beta_2 X_{12}^{\beta_1} X_{22}^{\beta_2-1}/(1+r)^2$$

$$+ \beta_0(\beta_1+\beta_2+1)(\beta_1+\beta_2)(X_{12}-X_{22})^{\beta_1+\beta_2-1}/(1+r). \quad (13)$$

But, if β_2 is negative then, since β_1 is negative, both terms of equation (13) are negative over the entire range $0 \leq X_{22} \leq X_{12}$. That is, f_2 attains an interior maximum over the interval, but the minimum must be at one of the end points. Substituting the endpoints into equation (11) shows $X_{22}=X_{12}$ to be the minimum.

In the first case, $\beta_1+\beta_2+1<0$ and $X_{22}=0$, the optimal value of f_2 is

$$f_2^*(X_{12}) = \beta_0 X_{12}^{\beta_1+\beta_2+1}/(1+r), \quad (14)$$

and the objective function for stage 3 is

$$f_3(X_{13}) = \min_{0 \leq X_{23} \leq X_{13}} \beta_0 X_{13}^{\beta_1} X_{23}^{\beta_2+1}/(1+r)^3 \quad (15)$$

$$+ \beta_0(X_{13}-X_{23})^{\beta_1+\beta_2+1}/(1+r).$$

In the second case

$$f_2^*(X_{12}) = \beta_0 X_{12}^{\beta_1+\beta_2+1}/(1+r)^2, \quad (16)$$

and the objective function at stage 3 is

$$f_3(X_{13}) = \min_{0 \leq X_{23} \leq X_{13}} \beta_0 X_{13}^{\beta_1} X_{23}^{\beta_2+1}/(1+r)^3 \quad (17)$$

$$+ \beta_0(X_{13}-X_{23})^{\beta_1+\beta_2+1}/(1+r)^t.$$

The objective functions at equations (15) and (17) clearly yield the same solutions as equation (11). That is if $\beta_1+\beta_2+1 \leq 0$ then $X^*_{23} = 0$ and if $\beta_1+\beta_2+1 > 0$ then $X^*_{23} = 0$ and if $\beta_1+\beta_2+1 > 0$ then $X^*_{23} = X_{13}$.

Since all the other stages in the problem are similar, the solution to the problem is clear. If $\beta_1+\beta_2+1 \leq 0$, the entire program requirement, V, should be produced in period 1; and if $\beta_1+\beta_2+1 > 0$, the entire program requirement should be produced in period T. This behavior is in dramatic contrast to the results reported in [11] which does tend to conform to observed contractor behavior where β_2 is assumed to be positive. Similar results can be derived by applying continuous time optimal control theory models and examining their second order conditions. The general relations among these models and others are explored in [16].

CONCLUSION

The combined influence of improvement and production rate on cost is still a topic that requires much additional research. Most previous modeling attempts must be interpreted with extreme care because they suffer from severe statistical problems. If there were no data problems (e.g., engineering change orders), and production rate could be measured accurately, the regression equation may be a valid tool for prediction purposes. However, the attempt to make policy statements using the estimates from equation (2) alone is futile.

In terms of production planning, a contractor certainly would not plan to operate at a less than optimal rate if cost minimization is induced by the contract. In many actual production programs planned production has exceeded actual production. Only the inability of the contractor to deliver on cost and on schedule has resulted in decreased production. In view of the results of this paper, it would seem that the above phenomena could imply that contractors are producing at greater than optimal rates given fixed facilities, so that diminishing returns to the variable resources exist throughout the program after some start-up period. Again it must be noted that this does <u>not</u> imply that the average cost per unit would have to rise as output rate is increased. The improvement effect could dominate the rate effect and average cost per unit could decline.

Finally, it can be argued that the model is flawed because it doesn't explicitly consider capacity constraints or progress billing. This is irrelevant for the results presented in this paper. Equation (2) is presently being used by the Air Force. Therefore, statements about the appropriateness of equation (2) are statements about Air Force planning. In any case, the results of this paper are valid even in the absence of a capacity constraint. Using the estimation procedure presented in [15], we have obtained estimates with the appropriate signs for the F-4, F-102, C-141, T-38, and Blackhawk programs.

The type of analysis presented in this paper does not solve the government's problem of how to ascertain the cost impact of production rate changes. However, it does suggest that models based on equation (2) alone are not the answer to the problem. These results should be noted and studied since the government has developed and most likely will use equations such as those analyzed in this paper.

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